



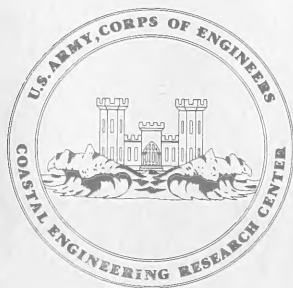
Interpretation of Wave Energy Spectra

by

Edward F. Thompson

COASTAL ENGINEERING TECHNICAL AID NO. 80-5

JULY 1980



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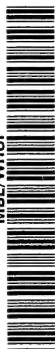
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| REPORT DOCUMENTATION PAGE | | READ INSTRUCTIONS BEFORE COMPLETING FORM |
|--|-----------------------|--|
| 1. REPORT NUMBER CETA 80-5 | 2. GOVT ACCESSION NO. | 3. RECIPIENT'S CATALOG NUMBER |
| 4. TITLE (and Subtitle) INTERPRETATION OF WAVE ENERGY SPECTRA | | 5. TYPE OF REPORT & PERIOD COVERED Coastal Engineering Technical Aid |
| | | 6. PERFORMING ORG. REPORT NUMBER |
| 7. AUTHOR(s) Edward F. Thompson | | 8. CONTRACT OR GRANT NUMBER(s) |
| 9. PERFORMING ORGANIZATION NAME AND ADDRESS Department of the Army Coastal Engineering Research Center (CERRE-CO) Kingman Building, Fort Belvoir, Virginia 22060 | | 10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS A31463 |
| 11. CONTROLLING OFFICE NAME AND ADDRESS Department of the Army Coastal Engineering Research Center Kingman Building, Fort Belvoir, Virginia 22060 | | 12. REPORT DATE July 1980 |
| | | 13. NUMBER OF PAGES 21 |
| 14. MONITORING AGENCY NAME & ADDRESS (If different from Controlling Office) | | 15. SECURITY CLASS. (of this report) UNCLASSIFIED |
| | | 15a. DECLASSIFICATION/DOWNGRADING SCHEDULE |
| 16. DISTRIBUTION STATEMENT (of this Report) Approved for public release; distribution unlimited. | | |
| 17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report) | | |
| 18. SUPPLEMENTARY NOTES | | |
| 19. KEY WORDS (Continue on reverse side if necessary and identify by block number) | | |
| Spectral analysis Wave gages Wave hindcast Wave energy spectra Wave height Wave period | | |
| 20. ABSTRACT (Continue on reverse side if necessary and identify by block number) | | |
| <p>Guidelines for interpreting nondirectional wave energy spectra are presented. A simple method is given for using the spectrum to estimate a significant height and period for each major wave train in most sea states. The method allows a more detailed and accurate description of ocean surface waves than that given by a single significant height and period, yet it eliminates much of the formidable detail of a full spectrum. An example problem illustrating application of the method is presented. Spectral analysis and display techniques, and the natural variation of spectra in space and time, are discussed.</p> | | |


PREFACE

This report presents guidelines for interpreting nondirectional wave energy spectra. The guidelines apply to spectra derived from both wave gage measurements and from numerical wave hindcasting models. A method is provided for using the spectrum to estimate a significant height and period for each major wave train in a sea state, except major wave trains with nearly the same period and different directions cannot be distinguished. The method has undergone limited testing and has been applied to 7 station-years of gage data, but further testing in well-documented field situations is needed. The guidelines and method are consistent with but are more practical and explicit than the material in the Shore Protection Manual (SPM). The work was done under the wave measurement program of the U.S. Army Coastal Engineering Research Center (CERC).

This report was prepared by Edward F. Thompson, Hydraulic Engineer, under the general supervision of Dr. C.L. Vincent, Chief, Coastal Oceanography Branch. Helpful reviews by Dr. C.L. Vincent, Dr. D.L. Harris, P. Knutson, and P. Vitale are acknowledged. Dr. D. Esteva provided the data for Figure 6.

Comments on this publication are invited.

Approved for publication in accordance with Public Law 1966, 79th Congress, approved 31 July 1945, as supplemented by Public Law 1972, 88th Congress, approved 7 November 1963.


TED E. BISHOP
Colonel, Corps of Engineers
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CONVERSION FACTORS, U.S. CUSTOMARY TO METRIC (SI) UNITS OF MEASUREMENT

U.S. customary units of measurement used in this report can be converted to metric (SI) units as follows:

| Multiply | by | To obtain |
|--------------------|-------------------------|---|
| inches | 25.4 | millimeters |
| | 2.54 | centimeters |
| square inches | 6.452 | square centimeters |
| cubic inches | 16.39 | cubic centimeters |
| feet | 30.48 | centimeters |
| | 0.3048 | meters |
| square feet | 0.0929 | square meters |
| cubic feet | 0.0283 | cubic meters |
| yards | 0.9144 | meters |
| square yards | 0.836 | square meters |
| cubic yards | 0.7646 | cubic meters |
| miles | 1.6093 | kilometers |
| square miles | 259.0 | hectares |
| knots | 1.852 | kilometers per hour |
| acres | 0.4047 | hectares |
| foot-pounds | 1.3558 | newton meters |
| millibars | 1.0197×10^{-3} | kilograms per square centimeter |
| ounces | 28.35 | grams |
| pounds | 453.6 | grams |
| | 0.4536 | kilograms |
| ton, long | 1.0160 | metric tons |
| ton, short | 0.9072 | metric tons |
| degrees (angle) | 0.01745 | radians |
| Fahrenheit degrees | 5/9 | Celsius degrees or Kelvins ¹ |

¹To obtain Celsius (C) temperature readings from Fahrenheit (F) readings, use formula: $C = (5/9) (F - 32)$.

To obtain Kelvin (K) readings, use formula: $K = (5/9) (F - 32) + 273.15$.

SYMBOLS AND DEFINITIONS

| | |
|--------------------------|---|
| a_j | wave component amplitude |
| d | water depth |
| E | sum of all E_j |
| $E_j, E(f_j)$ | spectral energy density values |
| E_{max} | energy density for the highest spectral peak |
| f_j | frequency of spectral component in hertz |
| f_p | frequency corresponding to the highest spectral peak |
| f_{pi} | frequency corresponding to the i th highest spectral peak |
| g | acceleration due to gravity |
| H_s | significant wave height corresponding to the full spectrum |
| H_{si} | significant wave height corresponding to the i th highest spectral peak |
| K_1, K_2 | indices representing the lower and upper bounds of the highest spectral peak |
| K_{1i}, K_{2i} | indices representing the lower and upper bounds of the i th highest spectral peak |
| N | total number of spectral frequency components |
| $S_j, S(f_j)$ | spectral energy values |
| S_{pi} | energy contained in i th highest spectral peak |
| t | time |
| T_p | period corresponding to the highest spectral peak |
| T_{pi} | period corresponding to the i th highest spectral peak |
| $\Delta f, (\Delta f)_j$ | frequency bandwidth represented by each spectral energy density (in hertz) |
| η | sea-surface elevation referenced to local mean water level |
| ϕ_j | phase of spectral component |
| ω_j | frequency of spectral component (in radians per second) |

INTERPRETATION OF WAVE ENERGY SPECTRA

by
Edward F. Thompson

I INTRODUCTION

The ocean usually has more than one independent train of waves propagating along its surface in U.S. coastal areas. The common practice of using a single significant height and period for a sea state can be misleading because no indication is given to the existence or characteristics of other trains. On the other hand, an estimate of the wave energy spectrum provides more information than is generally used in coastal engineering. The spectrum can be reduced to estimates of significant wave height and period for all major wave trains present. A knowledge of these characteristics for major wave trains is often important to coastal engineers.

Spectra are becoming widely available through various field wave measurement programs, laboratory tests with programable wave generators, and numerical wave hindcasting projects. Because of the availability and applications of spectra, practicing coastal engineers should become familiar with spectra and their interpretation.

II. PRIMARY GOAL OF SPECTRAL ANALYSIS

A fundamental parameter for characterizing a wave field is some measure of the periodicity of the waves. For many years a significant period, which could be subjectively estimated in various ways, was used. However, the ocean surface often has waves characterized by several distinct periods occurring simultaneously. A record of the variation of sea-surface elevation with time, commonly called a *time series*, frequently appears confusing and is difficult to interpret.

Developments in computer technology and in mathematical analysis of time series have provided a practical approach to an objective, more comprehensive analysis of periodicity in wave records. The approach is to express the time series as a sum of periodic functions with different frequency, amplitude, and phase. The simplest functions to apply are the trigonometric sine and cosine functions. Thus, the time series of sea-surface deviations from the mean surface, $\eta(t)$, is expressed by equation (3-11) in the Shore Protection Manual (SPM) (U.S. Army, Corps of Engineers, Coastal Engineering Research Center, 1977) as

$$\eta(t) = \sum_{j=1}^n a_j \cos (\omega_j t - \phi_j) \quad (1)$$

where

a_j = amplitude

ω_j = frequency in radians per second

ϕ_j = phase

t = time

Frequency is often expressed in terms of hertz units where one hertz is equal to one cycle per second. One hertz is also equivalent to 2π radians per second. If the symbol f_j denotes frequency in hertz, then $2\pi f_j = \omega_j$.

The amplitudes, a_j , computed for a time series, give an indication of importance of each frequency, f_j . The sum of the squared amplitudes is related to the variance of sea-surface elevations in the original time series (eq. 3-12 in the SPM) and hence to the potential energy contained in the wavy sea surface. Because of this relationship, the distribution of squared amplitudes as a function of frequency can be used to estimate the distribution of wave energy as a function of frequency. This distribution is called the *energy spectrum* and is often expressed as

$$(E_j)(\Delta f)_j = \frac{a_j^2}{2} = S_j \quad (2)$$

where $E_j = E(f_j)$ is the energy density in j th component of energy spectrum, $(\Delta f)_j$ the frequency bandwidth in hertz (difference between successive f_j), and $S_j = S(f_j)$ energy in j th component of energy spectrum. Equation (2) is similar to equation (3-15) in the SPM.

An energy spectrum computed from an ocean wave record is plotted in Figure 1. Frequencies associated with large values of energy density (or large values of $a_j^2/[2(\Delta f)_j]$; see eq. 2) represent dominant periodicities in the original time series. Frequencies associated with small values of $a_j^2/[2(\Delta f)_j]$ are usually unimportant. It is common for ocean wave spectra to show two or more dominant periodicities (Fig. 1).

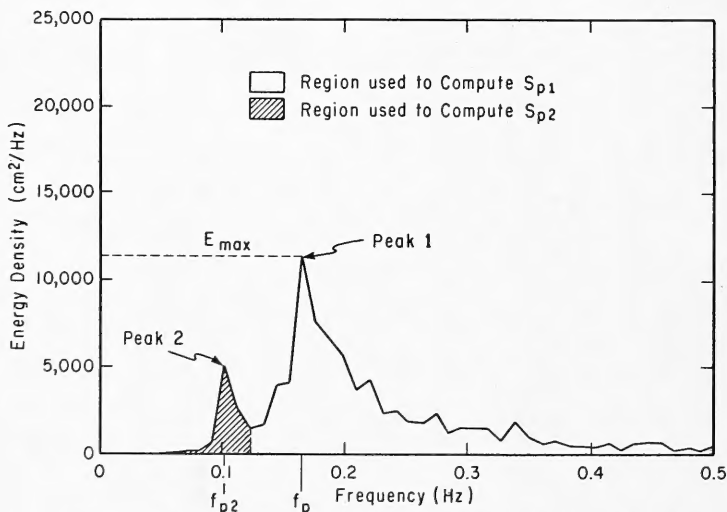


Figure 1. Spectrum for Wrightsville Beach, North Carolina, 0700 e.s.t., 12 February 1972; $H_g = 4.2$ feet (128 centimeters), $\Delta f = 0.01074$ hertz, and depth = 17.7 feet (5.4 meters).

The primary goal of spectral analysis is to objectively identify all important frequencies in a wave record. Since wave period in seconds is equal to the reciprocal of frequency in hertz, important wave periods are also identified.

III. PRACTICAL LIMITATIONS OF SPECTRAL ANALYSIS

1. Calculation Procedures.

The appearance of a spectrum can be noticeably influenced by the methods used for calculation and display, neither of which is standardized in coastal and ocean engineering activities.

Spectra are computed from both digital wave records and analog records for which an assortment of analog spectral analysis devices exists. A spectrum is computed from a digital wave record by either (a) computing the fourier transform of the autocovariance function of the record, or (b) computing the fourier transform of the record directly from the record using the fast fourier transform (FFT) approach (see Harris, 1974, for further detail). Both of these algorithms are used in conjunction with wave records subjected to various filters and smoothing functions before analysis. Further, some form of smoothing, averaging, or summing is often applied to the computed spectral components.

Different methods for calculating a spectrum will produce slightly different estimates of the spectrum when applied to a particular wave record. Major differences in the height of the spectral peak were shown by Wilson, Chakrabarti, and Snider (1974) when different approximations to the autocovariance function were used and different smoothing functions were applied to the spectrum of a field wave record (Fig. 2). Major differences in the height of the spectral peak and energy levels between peaks were noted by Harris (1974) when a time series composed of three superimposed sinusoidal waves was analyzed by several accepted methods (Fig. 3).

Hindcast wave energy spectra are computed by estimating atmospheric input of energy to the sea surface and redistribution of energy within a spectrum. The estimates are based on a series of equations derived from the physics of air-sea interaction and waves. The quality and characteristics of hindcast spectra are a function of the model used to perform the calculations (Resio and Vincent, 1979) as well as the accuracy of the input wind field.

Spectra obtained from either measurements or hindcasts are also limited by the resolution of the computation technique. The energy density and frequency at a spectral peak can be noticeably distorted if the frequency bandwidths, $(\Delta f)_j$, are not small enough to permit clear definition of major peaks.

2. Display Formats.

The appearance of a spectrum can be strongly affected by the display format used. Harris (1972) showed five often used formats for plotting spectra (Fig. 4). Each format alters the appearance of the spectrum. Format E shifts the relative magnitudes of spectral peaks enough that the second highest peak in formats A, B, C, and D becomes the highest peak in format E. A and C are the two most frequently used formats.

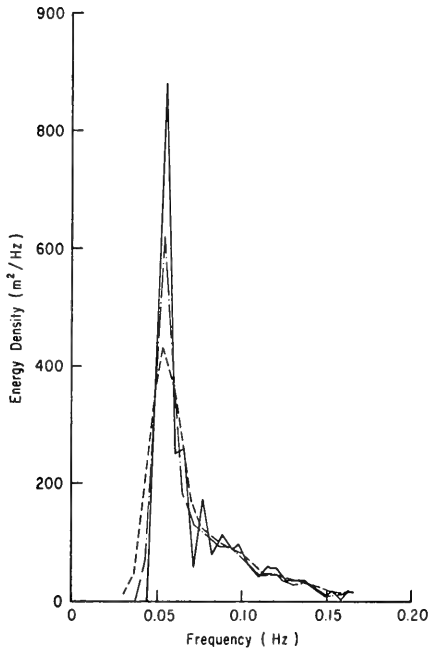


Figure 2. Energy spectra for wave record at 0400 e.s.t., 17 March 1968, from a weather ship in the North Atlantic, computed with different approximations to the autocovariance function and different spectral smoothing functions (after Wilson, Chakrabarti, and Snider, 1974).

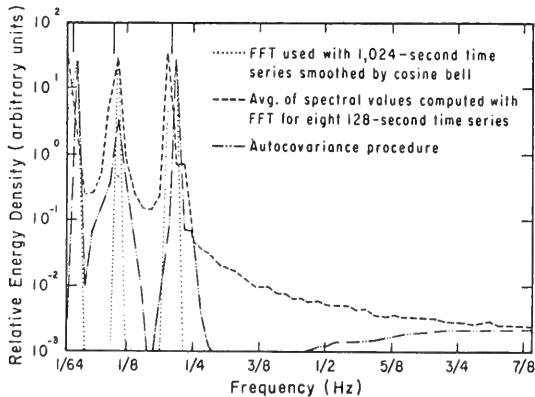


Figure 3. Energy spectra for record composed of three superimposed sinusoidal waves. Simulated frequencies indicated by vertical lines from top of graph (after Harris 1974).

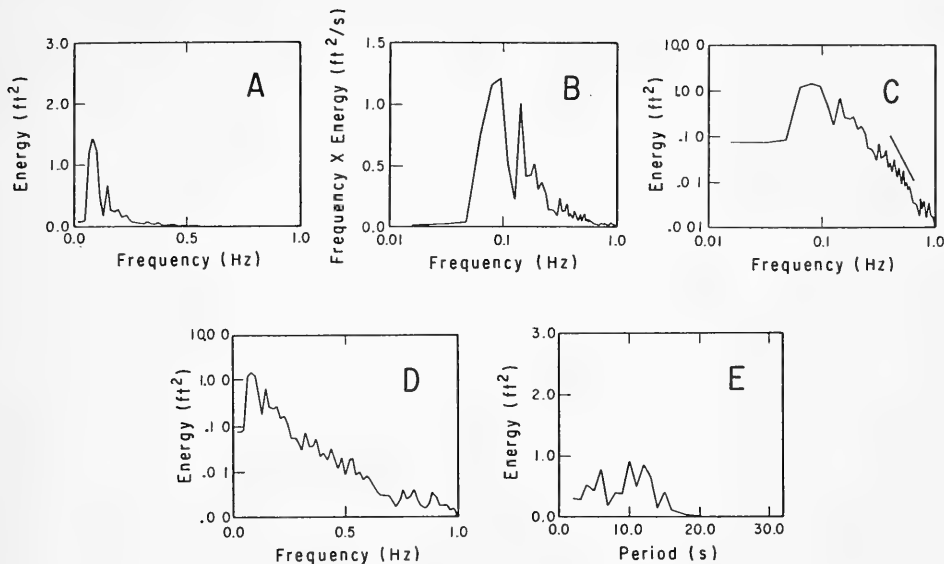


Figure 4. Five formats frequently used in displaying wave energy spectra. The actual spectrum is identical in all five graphs. The frequency bandwidth, $(\Delta f)_j$, is constant for all j so that energy values and energy density values differ by a constant factor (see eq. 2). The plots would look the same if energy were replaced by energy density, but the vertical scale would change (from Harris, 1972).

3. Natural Variability.

Wave energy spectra are naturally variable simply because they are based on a finite length record of a wave field which varies in time and space. Spectra computed for successive records of a relatively stationary wave field are never identical and often differ noticeably. The magnitude of spectral variation in time is illustrated by spectra derived at 2-hour intervals from two pressure gages along the southern California coast (Fig. 5). The significant wave height is nearly constant in the figure.

Spatial variation of the spectrum over short alongshore distances in shallow water is also shown in Figure 5. Each spectrum in the top row of the figure can be compared to the spectrum immediately below it to see variations between spectra from two gages 80 feet (24 meters) apart. In this figure, spatial variations are smaller than temporal variations. Spatial variations would be expected to be greater if the gages were farther apart or the water depth varied between measurement points. Variations between spectra from gages situated along a line perpendicular to shore are shown in Figure 6. The spatial variations are more prominent in this figure than in Figure 5.

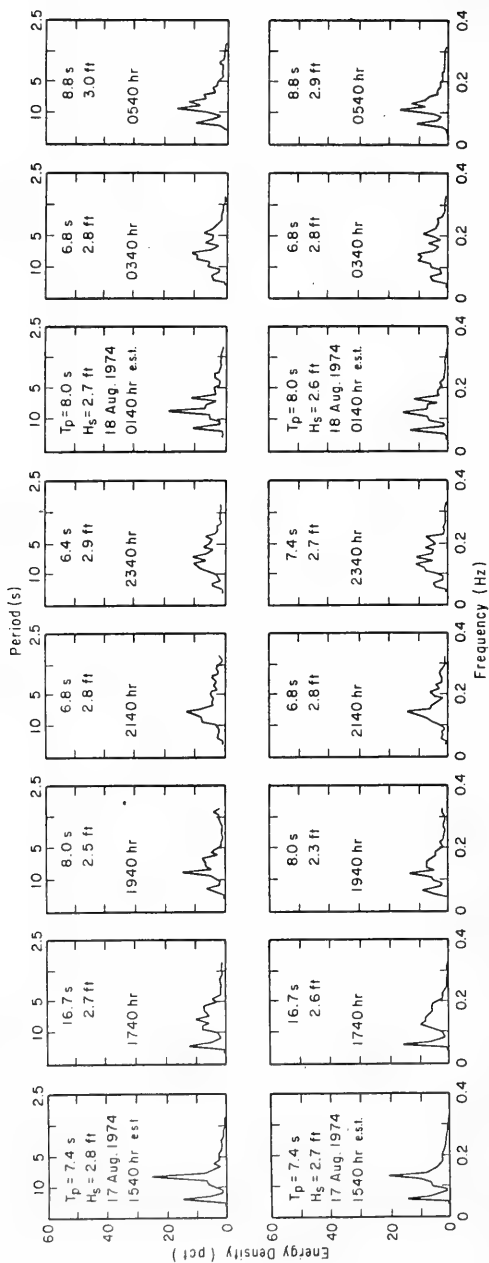


Figure 5. Wave energy spectra from bottom-mounted pressure gages at Channel Islands Harbor, California. Spectra in top row from gage 1; in bottom row from gage 2. Gages were 80 feet (24 meters) apart in water depth of 20 feet (6.1 meters). Spectra have been compensated for hydrodynamic attenuation due to submergence of the gage.

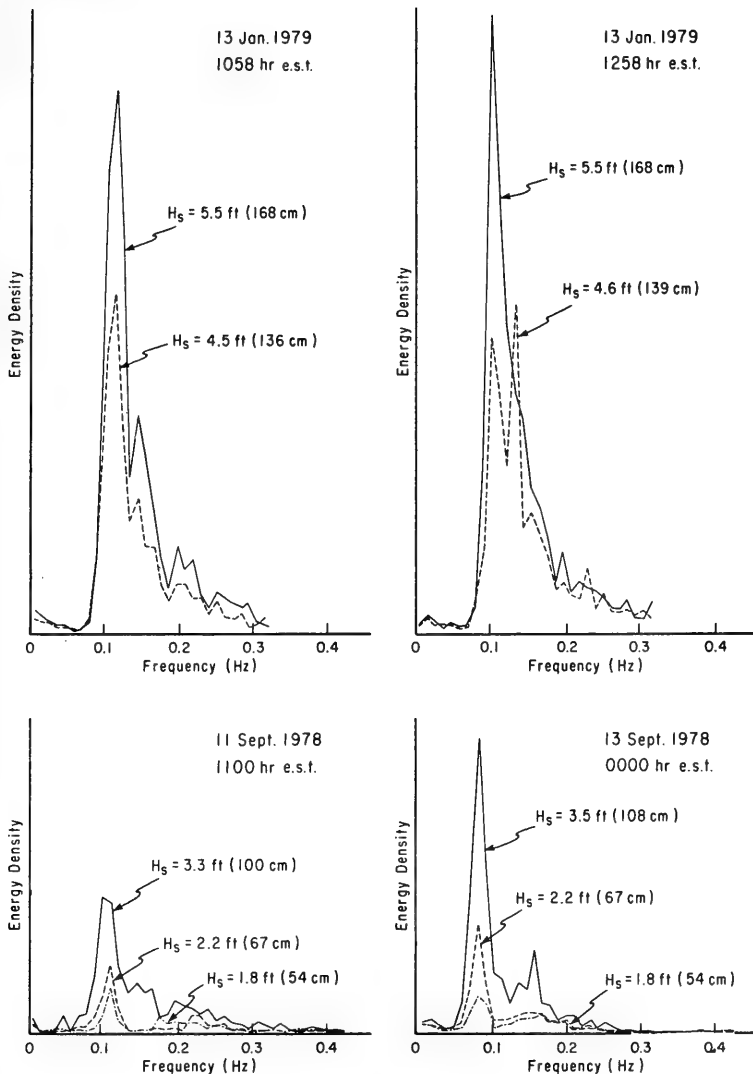


Figure 6. Wave energy spectra from pier-mounted continuous wire gages at the CERC Field Research Facility (FRF) near Duck, North Carolina, showing variation along a line perpendicular to shore. Solid lines represent a gage at the seaward pier end (depth 29 feet or 8.8 meters); dashlines represent a gage 480 feet (146 meters) from the seaward pier end (depth 22 feet or 6.6 meters); dot-dash lines represent a gage 840 feet (256 meters) from the pier end (depth 17 feet or 5.1 meters).

IV. INTERPRETATION OF SPECTRA

Because of natural variability in the spectrum and artificial variability induced by analysis and display techniques, the spectrum should never be interpreted as an exact representation of energy density versus frequency for a wave field. However, certain major features of the spectrum are consistent and meaningful.

1. Highest Spectral Peak.

a. Frequency and Period. Frequency corresponding to the highest spectral peak, f_p , is usually a reliable measure of the dominant wave frequency; f_p is shown in Figure 1. Period corresponding to the highest spectral peak, T_p , is equal to the reciprocal of f_p and is usually a good estimate of the dominant wave period.

b. Energy and Significant Wave Height. Energy contained in the highest peak, S_{p1} , is defined as the total energy in the vicinity of the highest peak.

$$S_{p1} = \sum_{j=K_1+1}^{K_2} E_j(\Delta f)_j \quad (3)$$

where K_1 and K_2 are indices representing the lower and upper bounds of the main peak. The upper and lower bounds sometimes represent a broad range of frequencies (see Fig. 1). S_{p1} is relatively consistent, and is less influenced than the magnitude of the highest peak by data collection procedures, by analysis and summarization procedures, and by temporal and spatial variation.

Some spectral analysis procedures are designed so that $(\Delta f)_j = \Delta f$ is a constant for all j , which leads to

$$S_{p1} = (\Delta f) \sum_{j=K_1+1}^{K_2} E_j \quad (3a)$$

Significant wave height corresponding to highest spectral peak H_{s1} , is an estimate of the significant height for the wave train represented by the highest spectral peak. It is computed by the relationship

$$H_{s1} = 4\sqrt{S_{p1}} \quad (4)$$

Energy density at the highest spectral peak, E_{max} , can be an indicator of how well focused the wave energy is in frequency. Although this parameter is variable, major differences in E_{max} (on the order of 50 percent) between spectra analyzed by the same method can be meaningful. E_{max} is shown in Figure 1.

2. Major Secondary Spectral Peaks.

a. Identification of Major Secondary Peaks. Major secondary spectral peaks are often indicative of independent secondary wave trains characterized by different heights, periods, and directions than the train represented by the main peak (examples are given in McClenan and Harris, 1975). Identification of major secondary spectral peaks involves some subjective judgment, but an objective

test for major secondary peaks has been developed and used at CERC. The test is applied to the difference in energy density between a spectral peak and the preceding and following spectral valleys. If that difference exceeds 3 percent of the total of all spectral energy density values, E , then the peak is considered major. Details of the procedure with a computerized version are given in Thompson (1980). The procedure was applied to the spectrum in Figure 1, computed from an ocean wave record, and two major peaks were identified. The procedure has been applied to 7 station-years of shallow-water spectra by Thompson (1980) to show that two-thirds of the ocean and gulf coast spectra have more than one major peak.

If two independent trains have nearly the same frequency but different directions, they cannot be identified by the method presented in this report. However, an analogous method could be developed for use with directional wave spectra to identify all wave trains which have distinct frequencies or directions. Directional spectra which give estimates of the distribution of wave energy density as a function of both frequency and direction are becoming increasingly available, mainly from spectral hindcasting models but also from improved measurement devices. A directional spectrum obtained by the Data Buoy Office of the National Oceanic and Atmospheric Administration (NOAA) from an experimental large discus buoy (Burdette, Steele, and Trampus, 1978) is shown in Figure 7. The figure indicates a concentration of low-frequency wave energy coming from the north-northeast and a concentration of high-frequency energy coming from the quadrant between east and south.

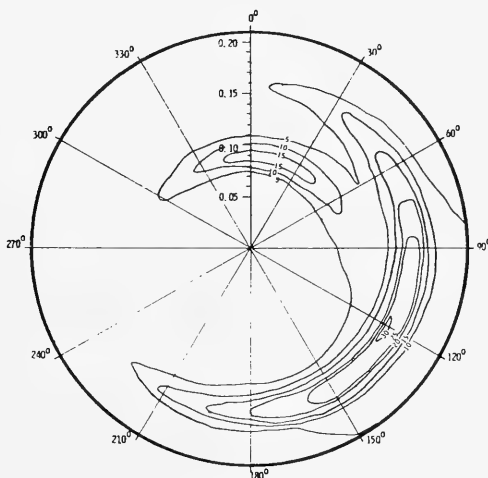


Figure 7. Directional spectrum obtained in the Atlantic Ocean 68 miles (110 kilometers) east of Jacksonville, Florida, at 1900 e.s.t., 30 March 1977. Contour values are in units of 0.001 meter squared per hertz per degree. The radial coordinate is frequency in hertz; the azimuthal coordinate is direction from which energy is coming (from Burdette, Steele, and Trampus, 1978).

Identification of major spectral peaks can also be a useful step toward approximating water velocities and accelerations from a complex record of sea-surface elevations, although procedures for doing this are not well established. Velocities and accelerations could be estimated for each peak separately, using equations in the SPM (Sec. 2.234); then they could be added together to give estimates of resultant velocity and acceleration. If direction estimates are available, the velocity and acceleration components should be added vectorially. An alternative approach is to use the nondirectional spectrum to estimate a velocity spectrum (Harris, 1972).

Occasionally, spectra have major secondary peaks which do not represent independent wave trains. In particular, when waves are very steep or in very shallow water the spectrum will often have secondary peaks at frequencies which are integral multiples of the dominant frequency. The secondary peaks indicate that additional frequencies are needed to represent the nonsinusoidal wave profiles (the peaks do not necessarily represent independent wave trains). When a major secondary peak appears at twice the dominant frequency, its source can usually be identified by referring to a plot of the time series from which the spectrum was computed. The time series is expected to show wave profiles which are clearly nonsinusoidal if the major secondary peak is nonindependent. Coarse guidelines for when nonindependent secondary peaks may occur are

$$\text{steepness: } \frac{H_s}{gT_p^2} > 0.008 \quad (5)$$

or

$$\text{relative depth: } \frac{d}{gT_p^2} < 0.01 \quad (6)$$

In cases where both steepness and relative depth approach the above guidelines, nonindependent peaks may also be evident.

b. Frequency and Period. Frequency and period corresponding to major secondary spectral peaks, f_{pi} and T_{pi} , usually indicate secondary frequencies and periods at which major amounts of energy are present. f_{p2} , corresponding to the second highest spectral peak, is shown in Figure 1. Equations (5) and (6) can be helpful in identifying cases where major secondary peaks do not represent independent energy concentrations.

c. Energy and Significant Wave Height. Energy and significant wave height corresponding to major secondary spectral peaks, S_{pi} , and H_{si} , indicate the relative importance of secondary peaks. Since major secondary peaks often represent independent wave trains, S_{pi} and H_{si} can be estimates of the energy and significant height of secondary trains. A method used at CERC to estimate S_{pi} is to partition the spectrum at its lowest point between every pair of adjacent major peaks (Fig. 1). The energy between partitions is then totaled by

$$S_{pi} = \sum_{j=K_{1i}+1}^{K_{2i}} E_j(\Delta f)_j \quad (7)$$

where K_{1i} is the index representing the lower bound of the peak, and K_{2i} the index representing the upper bound of the peak. If $(\Delta f)_j = \Delta f = \text{constant}$, then

$$S_{pi} = (\Delta f) \sum_{j=K_{1i}+1}^{K_{2i}} E_j \quad (7a)$$

Significant height is estimated from the total energy assigned to each peak by an equation similar to equation (4).

$$H_{gi} = 4 \sqrt{S_{pi}} \quad (8)$$

3. Example Problem.

This problem illustrates a method for estimating peak frequencies, periods, and significant heights for a spectrum with two major peaks.

GIVEN: Wave spectrum from Huntington Beach, California, for which the significant height is $H_g = 5.7$ feet (175 centimeters) (Fig. 8 and Table). Energy density is expressed as a percent of the sum of energy densities for all f_j listed in Table. $H_g = 5.7$ feet is based on 100 percent of the energy in the spectrum.

FIND: Estimate a separate significant wave height and peak period for each wave train indicated by the spectrum.

SOLUTION: To identify major spectral peaks, the difference in energy density between peak 2 and the valley between peaks 1 and 2 is estimated along the vertical axis in Figure 8 or from Table (about 4.5 percent). Since this is greater than 3 percent, peaks 1 and 2 are accepted as major peaks. Several other peaks appear at frequencies higher than 0.15 hertz, but there is no other combination of peak and valley for which the difference in energy density exceeds 3 percent. Therefore, peaks 1 and 2 are the only major peaks. Frequencies for peaks 1 and 2 are estimated along horizontal axis or from Table:

$$f_p = f_8 = 0.081 \text{ Hz}$$

$$f_{p2} = f_{13} = 0.135 \text{ Hz}$$

Since f_{p2} is not integral multiple of f_p , assume that peaks 1 and 2 represent independent wave trains. The reciprocal of each peak frequency gives peak period.

$$T_p = \frac{1}{f_p} = 12.3 \text{ s}$$

$$T_{p2} = \frac{1}{f_{p2}} = 7.4 \text{ s}$$

Compute total energy density, E , in the full spectrum by combining equations (3a) and (4) and rearranging to get

$$\begin{aligned} E &= \sum_{j=1}^{100} E_j = \left(\frac{H_g}{4} \right)^2 \left(\frac{1}{\Delta f} \right) \\ &= \left(\frac{5.7 \text{ ft}}{4} \right)^2 \left(\frac{1}{0.01074 \text{ Hz}} \right) \\ &= 192 \text{ ft}^2/\text{Hz} \end{aligned}$$

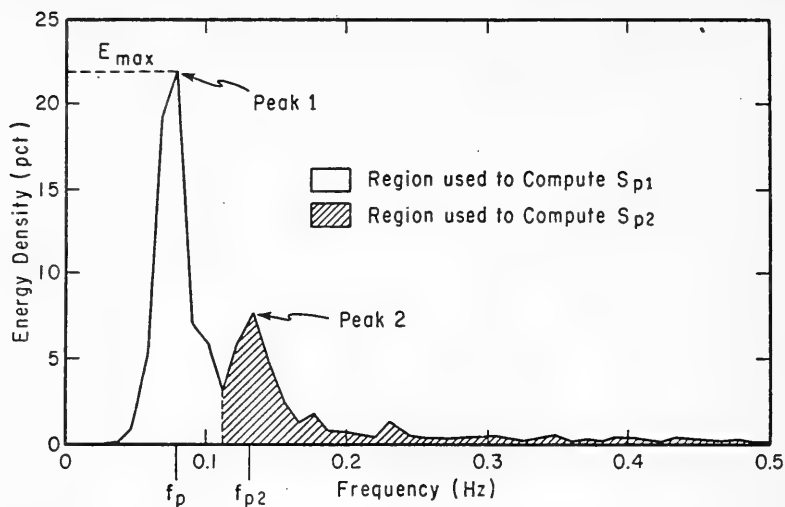


Figure 8. Spectrum for Huntington Beach, California, 0640 e.s.t., 24 August 1972; $H_g = 5.7$ feet (175 centimeters), $\Delta f = 0.01074$ hertz, and water depth = 26 feet (7.8 meters).

Table. Spectrum for Huntington Beach, California, 0640 e.s.t., 24 August 1972.¹

| j | f_j (Hz) | E_j (pct) | j | f_j (Hz) | E_j (pct) |
|----|---------------|----------------|-----------|---------------|----------------|
| 1 | 0.005 | 0 | 25 | 0.264 | 0.4 |
| 2 | 0.016 | 0 | 26 | 0.274 | 0.3 |
| 3 | 0.027 | 0 | 27 | 0.285 | 0.3 |
| 4 | 0.038 | 0.2 | 28 | 0.296 | 0.4 |
| 5 | 0.049 | 1.1 | 29 | 0.307 | 0.4 |
| 6 | 0.059 | 5.1 | 30 | 0.317 | 0.2 |
| 7 | 0.070 | 19.3 | 31 | 0.328 | 0.1 |
| 8 | 0.081 | 21.9 | 32 | 0.339 | 0.3 |
| 9 | 0.092 | 7.0 | 33 | 0.349 | 0.4 |
| 10 | 0.102 | 5.9 | 34 | 0.360 | 0.1 |
| 11 | 0.113 | 3.1 | 35 | 0.371 | 0.2 |
| 12 | 0.124 | 5.9 | 36 | 0.382 | 0.1 |
| 13 | 0.135 | 7.6 | 37 | 0.392 | 0.3 |
| 14 | 0.145 | 4.9 | 38 | 0.403 | 0.3 |
| 15 | 0.156 | 2.6 | 39 | 0.414 | 0.2 |
| 16 | 0.167 | 1.3 | 40 | 0.425 | 0.1 |
| 17 | 0.178 | 1.8 | 41 | 0.435 | 0.4 |
| 18 | 0.188 | 0.7 | 42 | 0.446 | 0.3 |
| 19 | 0.199 | 0.7 | 43 | 0.457 | 0.2 |
| 20 | 0.210 | 0.5 | 44 | 0.468 | 0.1 |
| 21 | 0.221 | 0.4 | 45 | 0.478 | 0.2 |
| 22 | 0.231 | 1.3 | 46 | 0.489 | 0.1 |
| 23 | 0.242 | 0.6 | 47 | 0.500 | 0.1 |
| 24 | 0.253 | 0.4 | 48 to 100 | 0.5-1.0 | 2.2 |

¹ $H_g = 5.7$ feet (175 centimeters); $\Delta f = 0.01074$ Hertz.

Estimate the energy in peak 1 using equation (3a), the tabulated energy density (Table), and the above value of E to convert the sum of energy density from percent to foot squared.

$$\begin{aligned} S_{p1} &= (\Delta f) \sum_{j=4}^{11} E_j \\ &= (0.01074 \text{ Hz}) \left(\frac{63.6 \text{ pct}}{100 \text{ pct}} \right) (192 \text{ ft}^2/\text{Hz}) \\ &= 1.31 \text{ ft}^2 \end{aligned}$$

Similarly estimate the energy in peak 2 using equation (7a)

$$\begin{aligned} S_{p2} &= (\Delta f) \sum_{j=12}^{100} E_j \\ &= (0.01074 \text{ Hz}) \left(\frac{36.4 \text{ pct}}{100 \text{ pct}} \right) (192 \text{ ft}^2/\text{Hz}) \\ &= 0.75 \text{ ft}^2 \end{aligned}$$

Estimate significant height for peak 1 using equation (4).

$$\begin{aligned} H_{s1} &= 4\sqrt{S_{p1}} \\ &= 4\sqrt{1.31 \text{ ft}^2} = 4.6 \text{ ft} \end{aligned}$$

Note that H_{s1} is 1.1 feet lower than H_s based on the full spectrum. Similarly, significant height for peak 2 is estimated from equation (8)

$$H_{s2} = 3.5 \text{ ft}$$

H_s , H_{s1} and H_{s2} are related in this example by

$$H_s = \sqrt{H_{s1}^2 + H_{s2}^2}$$

V. INTERPRETATION OF SPECTRA FOR APPLICATIONS SENSITIVE TO SPECIFIC FREQUENCIES

The interpretation of spectra in terms of wave trains is appropriate for most coastal engineering work. However, certain engineering applications, especially applications in which resonance can occur, are highly sensitive to one frequency or a small range of frequencies. Estimates of how much energy can be expected at that frequency or range of frequencies are required. The estimates are obtained directly from the spectrum. Since the estimates are sensitive to data collection and analysis procedures, it is especially important that the procedures be optimum when such applications are intended or anticipated.

For frequency-sensitive applications, it is generally assumed that each frequency with nonzero energy represents an independent wave component, regardless of whether it is a spectral peak. Thus, in a resonance problem sensitive to frequency f_R , spectral energy at f_R is treated as an independent wave. If f_R corresponds to a major spectral peak, it would be appropriate (and conservative) to estimate the energy from equation (7). In a floating break-water problem where energy at frequencies lower than some cutoff frequency,

f_0 , will be transmitted through the breakwater, the transmitted energy is estimated from an incident spectrum by summing all energy at frequencies lower than f_0 , regardless of where the spectral peaks are located.

The assumption of independent spectral components seems adequate most of the time for such applications. However, for very steep waves or waves in very shallow water the assumption is often incorrect. Cases have been documented in which all major spectral components at frequencies higher than the main peak frequency are closely tied to the main peak and do not represent independent energy concentrations. The steepness and relative depth criteria in equations (5) and (6) can be used to indicate cases where the assumption of independent spectral components may be poor.

VI. SUMMARY

This report has presented guidelines for interpreting nondirectional wave energy spectra, including a simple method for identifying major spectral peaks and for estimating significant wave height, period, and energy for each major peak. Each major spectral peak is generally assumed to represent an independent wave train. Coarse guidelines are presented for identifying cases where major peaks do not represent independent trains. Spectral analysis and display techniques, and the natural variation of spectra in space and time, are discussed to show that the above method for interpreting spectra provides a relatively consistent description of general spectral characteristics. The method allows a more detailed and accurate description of ocean surface waves than that given by a single significant height and period, yet it eliminates much of the formidable detail of a full spectrum.

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